Friction
Today’s Objectives:
Students will be able to:

a) Determine the forces on a wedge.
b) Determine tension in a belt.

In-Class Activities:

- Check Homework, if any
- Reading Quiz
- Applications
- Analysis of a Wedge
- Analysis of a Belt
- Concept Quiz
- Group Problem Solving
- Attention Quiz
1. A wedge allows a ______ force P to lift a __________ weight W.
   A) (large, large)      B) (small, large)
   C) (small, small)      D) (large, small)

2. Considering friction forces and the indicated motion of the belt, how are belt tensions $T_1$ and $T_2$ related?
   A) $T_1 > T_2$   B) $T_1 = T_2$
   C) $T_1 < T_2$   D) $T_1 = T_2 e^\mu$
Applications

Wedges are used to adjust the elevation or provide stability for heavy objects such as this large steel pipe.

How can we determine the force required to pull the wedge out?

When there are no applied forces on the wedge, will it stay in place (i.e., be self-locking) or will it come out on its own? Under what physical conditions will it come out?
Belt drives are commonly used for transmitting the torque developed by a motor to a wheel attached to a pump, fan or blower.

How can we decide if the belts will function properly, i.e., without slipping or breaking?
In the design of a band brake, it is essential to analyze the frictional forces acting on the band (which acts like a belt).

How can you determine the tension in the cable pulling on the band?

Also from a design perspective, how are the belt tension, the applied force P and the torque M, related?
WEDGES

The wedge is a simple machine which may be used to transform horizontal input forces into vertical output forces.

– The angle of friction may be used to simplify the solution.
A wedge is a simple machine in which a small force $P$ is used to lift a large weight $W$.

To determine the force required to push the wedge in or out, it is necessary to draw FBDs of the wedge and the object on top of it.

It is easier to start with a FBD of the wedge since you know the direction of its motion.

Note that:
- a) the friction forces are always in the direction opposite to the motion, or impending motion, of the wedge;
- b) the friction forces are along the contacting surfaces; and,
- c) the normal forces are perpendicular to the contacting surfaces.
Next, a FBD of the object on top of the wedge is drawn. Please note that:

a) at the contacting surfaces between the wedge and the object the forces are equal in magnitude and opposite in direction to those on the wedge; and, b) all other forces acting on the object should be shown.

To determine the unknowns, we must apply EomE, \( \sum F_x = 0 \) and \( \sum F_y = 0 \), to the wedge and the object as well as the impending motion frictional equation, \( F = \mu_s N \).
Now of the two FBDs, which one should we start analyzing first?

We should start analyzing the FBD in which the number of unknowns are less than or equal to the number of E-of-E and frictional equations.
NOTE:
If the object is to be lowered, then the wedge needs to be pulled out. If the value of the force $P$ needed to remove the wedge is positive, then the wedge is self-locking, i.e., it will not come out on its own.
Consider a flat belt passing over a fixed rough cylinder, as shown here.

The relationship between the tensions $T_L$ (larger tension) and $T_S$ (smaller tension) at the ends of the belt is given by:

$$\frac{T_L}{T_S} = e^{\pi \mu \beta / 180^\circ}$$

...where $e$ is the base of the natural logarithm & the Greek letter $\beta$ (beta) is the angle of contact between the belt and cylinder in degrees.
Most conveyor systems, many pieces of power transmission equipment, and some machine tools found on production lines today operate using flat belts. Modern belts are generally constructed in layers: an inner surface of leather (for friction), a central core of nylon (for strength), and an outer layer of polyester or rubber (to protect the nylon core). These belts are capable of transmitting 150 hp/in. of belt width at linear speeds of up to 20,000 ft/min (about 225 mph!). Because of the various layer materials and belt thicknesses and widths available, belt tensions may need to be adjusted as shown as strains, expressed in percent (see Sec. 3).
Belt Analysis

Consider a flat belt passing over a fixed curved surface with the total angle of contact equal to $\beta$ radians.

If the belt slips or is just about to slip, then $T_2$ must be larger than $T_1$ and the motion resisting friction forces. Hence, $T_2$ must be greater than $T_1$.

Detailed analysis (please refer to your textbook) shows that $T_2 = T_1 e^{\mu \beta}$ where $\mu$ is the coefficient of static friction between the belt and the surface. Be sure to use radians when using this formula!!
Consider a flat belt passing over a fixed curved surface with the total angle of contact equal to $\beta$ radians.

Since $d\theta$ is of infinitesimal size, $\sin(d\theta/2) = d\theta/2$ and $\cos(d\theta/2) = 1$. Also, the product of the two infinitesimals $dT$ and $d\theta/2$ may be neglected when compared to infinitesimals of the first order. As a result, these two equations become:

$$\mu \, dN = dT$$
Belt Analysis

\[ \sum F_x = 0; \quad T \cos \left( \frac{d\theta}{2} \right) + \mu dN - (T + dT) \cos \left( \frac{d\theta}{2} \right) = 0 \]

\[ \sum F_y = 0; \quad dN - (T + dT) \sin \left( \frac{d\theta}{2} \right) - T \sin \left( \frac{d\theta}{2} \right) = 0 \]

\[ \mu dN = dT \]

\[ dN = T \, d\theta \]

\[ \frac{dT}{T} = \mu \, d\theta \]

Integrating this equation between all the points of contact that the belt makes with the drum, and noting that \( T = T_1 \) at \( \theta = 0 \) and \( T = T_2 \) at \( \theta = \beta \), yields

\[ \int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^{\beta} d\theta \]

\[ \ln \frac{T_2}{T_1} = \mu \beta \]
Belt Analysis

\[
\int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^\beta d\theta \\
\ln \frac{T_2}{T_1} = \mu \beta
\]

Solving for \( T_2 \), we obtain

\[
T_2 = T_1 e^{\mu \beta}
\]

where

\( T_2, T_1 \) are belt tensions; \( T_1 \) opposes the direction of motion (or impending motion) of the belt measured relative to the surface, while \( T_2 \) acts in the direction of the relative belt motion (or impending motion); because of friction, \( T_2 > T_1 \).

\( \mu \) is the coefficient of static or kinetic friction between the belt and the surface of contact.

\( \beta \) is the angle of belt to surface contact, measured in radians.

\( e = 2.718 \ldots \), base of the natural logarithm.
Given: The crate weighs 300 lb and $\mu_s$ at all contacting surfaces is 0.3. Assume the wedges have negligible weight.

Find: The smallest force $P$ needed to pull out the wedge.

Plan:

1. Draw a FBD of the crate. Why do the crate first?
2. Draw a FBD of the wedge.
3. Apply the E-of-E to the crate.
4. Apply the E-of-E to wedge.
The FBDs of crate and wedge are shown in the figures. Applying the E-of-E to the crate, we get

\[ + \sum F_X = N_B - 0.3N_C = 0 \]
\[ \uparrow + \sum F_Y = N_C - 300 + 0.3N_B = 0 \]

Solving the above two equations, we get

\[ N_B = 82.57 \text{ lb} = 82.6 \text{ lb}, \quad N_C = 275.3 \text{ lb} = 275 \text{ lb} \]
Example

Applying the E-of-E to the wedge, we get

\[ \uparrow + \sum F_Y = N_D \cos 15^\circ + 0.3 N_D \sin 15^\circ - 275.2 = 0; \]

\[ N_D = 263.7 \text{ lb} = 264 \text{ lb} \]

\[ \rightarrow + \sum F_X = 0.3(263.7) + 0.3(263.7)\cos 15^\circ - 0.3(263.7)\cos 15^\circ - P = 0; \]

\[ P = 90.7 \text{ lb} \]
Concept Quiz

1. Determine the direction of the friction force on object B at the contact point between A and B.
   A) ←
   B) →
   C) ↑
   D) ↓

2. The boy (hanging) in the picture weighs 100 lb and the woman weighs 150 lb. The coefficient of static friction between her shoes and the ground is 0.6. The boy will ______ ?
   A) Be lifted up
   B) Slide down
   C) Not be lifted up
   D) Not slide down
Given: Blocks A and B weigh 50 lb and 30 lb, respectively.

Find: The smallest weight of cylinder D which will cause the loss of static equilibrium.

Plan:
Plan:

1. Consider two cases: a) both blocks slide together, and, b) block B slides over the block A.

2. For each case, draw a FBD of the block(s).

3. For each case, apply the E-of-E to find the force needed to cause sliding.

4. Choose the smaller $P$ value from the two cases.

5. Use belt friction theory to find the weight of block D.
Case a (both blocks sliding together):
\[ \uparrow + \sum F_Y = N - 80 = 0 \]
N = 80 lb

\[ + \sum F_X = 0.4 \times 80 - P = 0 \]
P = 32 lb
Case b (block B slides over A):

\[
\begin{align*}
\uparrow + \sum F_y &= N \cos 20^\circ + 0.6 N \sin 20^\circ - 30 = 0 \\
N &= 26.20 \text{ lb} \\
\rightarrow + \sum F_x &= -P + 0.6 (26.2) \cos 20^\circ - 26.2 \sin 20^\circ = 0 \\
P &= 5.812 \text{ lb}
\end{align*}
\]

Case b has the lowest P (case a was 32 lb) and thus will occur first. Next, using a frictional force analysis of belt, we get

\[
W_D = P e^{\mu \beta} = 5.812 e^{0.5 (\pi/2)} = 12.7 \text{ lb}
\]

A Block D weighing 12.7 lb will cause the block B to slide over the block A.
1. When determining the force $P$ needed to lift the block of weight $W$, it is easier to draw a FBD of ______ first.

A) The wedge  
B) The block  
C) The horizontal ground  
D) The vertical wall

2. In the analysis of frictional forces on a flat belt, $T_2 = T_1 e^{\mu \beta}$. In this equation, $\beta$ equals ______.

A) Angle of contact in degrees  
B) Angle of contact in radians  
C) Coefficient of static friction  
D) Coefficient of kinetic friction
The screw jack has a threaded stem that turns in a fixed base.

- As a force $P$ is applied to the handle, the stem turns and moves up (or down) lifting (lowering) the load $W$. 
Frictional Forces on Screws

Upward Impending Motion. Let us now consider the case of a square-threaded screw that is subjected to upward impending motion caused by the applied torsional moment $M$, Fig. 8–15.* A free-body diagram of the entire unraveled thread can be represented as a block as shown in Fig. 8–14a. The force $W$ is the vertical force acting on the thread or the axial force applied to the shaft, Fig. 8–15, and $M/r$ is the resultant horizontal force produced by the couple moment $M$ about the axis of the shaft. The reaction $R$ of the groove on the thread, has both frictional and normal components, where $F = \mu_s N$. The angle of static friction is $\phi_s = \tan^{-1}(F/N) = \tan^{-1}\mu_s$. Applying the force equations of equilibrium along the horizontal and vertical axes, we have

$$\sum F_x = 0; \quad M/r - R \sin (\phi_s + \theta) = 0$$

$$\sum F_y = 0; \quad R \cos (\phi_s + \theta) - W = 0$$

Eliminating $R$ from these equations, we obtain

$$M = rW \tan (\phi_s + \theta)$$
The screw can be analyzed by considering the screw thread as an inclined plane rolled on a cylinder.

- Inclination of the plane is given by:

\[
\tan \theta = \frac{p}{\pi d}
\]

...where \( p \) is the pitch of the thread and \( d \) the mean diameter of the threads.
The friction force $F_m$ and normal force $N_R$ have been replaced by their resulting $R$, which acts at angle $\phi_s$ (angle of static friction) from the normal $N_R$. 
Frictional Forces on Screws

Self-Locking Screw. A screw is said to be self-locking if it remains in place under any axial load $W$ when the moment $M$ is removed. For this to occur, the direction of the frictional force must be reversed so that $R$ acts on the other side of $N$. Here the angle of static friction $\phi_s$ becomes greater than or equal to $\theta$, Fig. 8–16d. If $\phi_s = \theta$, Fig. 8–16b, then $R$ will act vertically to balance $W$, and the screw will be on the verge of winding downward.

(a) Upward screw motion

(b) Self-locking screw ($\theta = \phi_s$) (on the verge of rotating downward)

(c) Downward screw motion ($\theta < \phi_s$)
Frictional Forces on Screws

Downward screw motion ($\theta > \phi_s$)

Self-locking screw ($\theta = \phi_s$) (on the verge of rotating downward)

Downward Impeding Motion. ($\phi_s > \theta$). If a screw is self-locking, a couple moment $M'$ must be applied to the screw in the opposite direction to wind the screw downward ($\phi_s > \theta$). This causes a reverse horizontal force $M'/r$ that pushes the thread down as indicated in Fig. 8–16c. Using the same procedure as before, we obtain
SQUARE-THREADED SCREWS: SCREW JACKS

- From the equations of equilibrium:

\[
\begin{align*}
\sum F_x &= 0 \quad F_x - R \sin (\theta + \phi_s) = 0 \quad \text{(a)} \\
\sum F_y &= 0 \quad R \cos (\theta + \phi_s) - W = 0 \quad \text{(b)}
\end{align*}
\]

Eliminating \( R \) between (a) and (b) gives:

\[
F_x = \frac{W \sin (\theta + \phi_s)}{\cos (\theta + \phi_s)}
\]

\[
F_x = W \tan (\theta + \phi_s)
\]
SQUARE-THREADED SCREWS:
SCREW JACKS

• To produce motion, the moment of the force $P$ on the handle of the screw jack must be equal to the moment of the force $F_x$ at the mean radius $d/2$ of the screw—both about the axis of the screw.

Thus:

\[ M = PL = F_x \frac{d}{2} \]
SQUARE-THREADED SCREWS: SCREW JACKS

- Substituting for $F_x$ from

$$F_x = W \tan(\theta + \phi_s)$$

...gives:

$$M = PL = W \frac{d}{2} \tan(\theta + \phi_s)$$

...where:

- $P$ is the force applied to the handle.
- $L$ the length of the handle.
- $W$ the load to be *lifted* by the jack screw.
- $\theta$ the inclination of the plane of the screw given by:

$$\tan \theta = \frac{p}{\pi d}$$

and $\phi_s$ the angle of friction.
SQUARE-THREADED SCREWS: SCREW JACKS

• To lower the load, the friction force is reversed:

\[ M = PL = W \frac{d}{2} \tan (\phi_s - \theta) \]

• If \( \phi_s \geq \theta \), then \( M \) is a positive number, and the screw is said to be self-locking.
  – The weight will remain in place, even if \( M \) is removed.

• If \( \phi_s < \theta \), then \( M \) is a negative number, and the weight will lower by itself, unless \( M \) continues to be applied.
Journal bearings are used to provide lateral support to rotating machines like shafts & axles.
- If the bearing is partially lubricated or not lubricated at all, the methods of this chapter may be applied.
The journal bearing shown supports a shaft that is rotating at a constant speed. To maintain the rotation, a torque or moment couple \( M \) must be applied. The load \( W \) and moment couple \( M \) cause the shaft to touch the bearing at \( A \). The shaft climbs up on the bearing until the forces and couple are in equilibrium.
The shaft touching the bearing at A gives rise to reaction $N_R$ and friction force $F_k$ as shown in the free-body diagram.
For equilibrium:

\[ \Sigma F_y = 0 \quad W = R \]
\[ \Sigma M_O = 0 \quad M = R r \sin \phi_k \]

For small angles of friction:

\[ \sin \phi_k \approx \tan \phi_k = \mu_k \]

...thus:

\[ M \approx R r \mu_k \]
Although most bearings are made of metal or plastic, a surprising number of industrial applications use wood bearings. The basic material, typically a dense, close-grained hardwood such as maple, is impregnated with a blend of oil lubricants and is then machined into the desired shapes, usually to tolerances between 0.002 and 0.005 in. The finished products (SB.10) are self-lubricating; perform well in gritty or abrasive environments; and are not harmed by water, mild acids, alkalis, and most caustic chemicals. For these reasons, wood bearings are well-suited to equipment that operates in harsh environments, such as those found in agriculture, food processing, paper-making, sewage treatment, and marine operations.
Consider a hollow rotating shaft whose end is in contact with a fixed surface, as shown.

Assume the force between the rotating shaft and fixed surface is distributed uniformly.

External torque $M$ required to cause slipping to occur is given by:

$$M = \mu_k P \frac{2(R_o^3 - R_i^3)}{3(R_o^2 - R_i^2)}$$

...where $P$ is the thrust force, $R_o$ the outside radius, and $R_i$ the inside radius of the hollow shaft.
Consider a hollow rotating shaft whose end is in contact with a fixed surface, as shown.

For a solid shaft of radius $R$:

$$M = \mu_k P \left(\frac{2}{3}\right) R$$

The largest torque, without slipping & transmitted by a disk clutch can be found from these equations, where $\mu_k$ has been replaced by $\mu_s$. 
Rolling resistance of railroad wheels on the rails is small since steel is very stiff. By comparison, the rolling resistance of the wheels of a tractor in a wet field is very large.
ROLLING RESISTANCE

• If a wheel is moved without slipping over a horizontal surface while supporting a load, a force is required to maintain uniform motion.
  – Thus, some kind of rolling resistance must be present.
ROLLING RESISTANCE

• The usual method for describing rolling resistance is based on deformation of the surface as shown:

Let resultant reaction between the surface and rollers act at point A.

From equilibrium:

\[ \sum M_A = 0 \quad Wa - Ph = 0 \]

As \( h \) is nearly equal to \( r \):

\[ Wa - Pr = 0 \]

\[ P = \frac{Wa}{r} \]
ROLLING RESISTANCE

• The usual method for describing rolling resistance is based on deformation of the surface as shown:

The distance $a$ is called the *coefficient of rolling resistance*. The values of $a$ are given in millimeters or inches. They vary substantially for the same material with different values of $W$ and $r$. 
Rolling Resistance

\[ Wa = P(r \cos \theta) \]

\[ \cos \theta \approx 1 \]

\[ Wa \approx Pr \]

\[ P \approx \frac{Wa}{r} \]

Rigid surface of contact  Soft surface of contact

Coefficient of rolling resistance
A 10-kg steel wheel shown in Fig. 8–26a has a radius of 100 mm and rests on an inclined plane made of soft wood. If $\theta$ is increased so that the wheel begins to roll down the incline with constant velocity when $\theta = 1.2^\circ$, determine the coefficient of rolling resistance.

\[
\sum F_y = 0; \\
- (98.1 \cos 1.2^\circ \text{ N})(a) + (98.1 \sin 1.2^\circ \text{ N})(100 \cos 1.2^\circ \text{ mm}) = 0
\]

\[
a = 2.09 \text{ mm}
\]
Homework 4 (Part 3)

- Problem 8.62
- Problem 8.75
- Problem 8.102

Due date: 1\textsuperscript{st} hour of Week 12
(for both Part 1 & Part 2 & Part 3)
<table>
<thead>
<tr>
<th>English</th>
<th>Turkish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frictional force</td>
<td>Sürtünme kuvveti</td>
</tr>
<tr>
<td>impending</td>
<td>Eli kulağında</td>
</tr>
<tr>
<td>tangent</td>
<td>teğet</td>
</tr>
<tr>
<td>Area of contact</td>
<td>Temas alanı</td>
</tr>
<tr>
<td>coefficient</td>
<td>Katsayı, sabite</td>
</tr>
<tr>
<td>slipping</td>
<td>kayma</td>
</tr>
<tr>
<td>apparent</td>
<td>Aşikar, belli, görülür</td>
</tr>
<tr>
<td>crate</td>
<td>Kasa, sandık</td>
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<tr>
<td>tipping</td>
<td>Yana yatma</td>
</tr>
<tr>
<td>dump</td>
<td>boşaltma</td>
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<tr>
<td>Vending machine</td>
<td>Satış otomatı</td>
</tr>
<tr>
<td>ladder</td>
<td>merdiven</td>
</tr>
<tr>
<td>wedge</td>
<td>kama</td>
</tr>
<tr>
<td>Square-threaded</td>
<td>Kare-kesit dişli</td>
</tr>
<tr>
<td>screw</td>
<td>vida</td>
</tr>
<tr>
<td>Self-locking</td>
<td>Kendiliğinden kilitlenen</td>
</tr>
<tr>
<td>turnbuckle</td>
<td>firdöndü</td>
</tr>
<tr>
<td>Flat-belt</td>
<td>Düz kayış</td>
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<tr>
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<td>V-kayış</td>
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<td>Şerit, kordon</td>
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<tr>
<td>drum</td>
<td>tambur</td>
</tr>
<tr>
<td>Pivot-bearing</td>
<td>Muyu yatak</td>
</tr>
<tr>
<td>Collar-bearing</td>
<td>Bilezikli yatak</td>
</tr>
<tr>
<td>Journal-bearing</td>
<td>Kaymalı yatak</td>
</tr>
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<td>Sanding machine</td>
<td>Taşlama makinesi</td>
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<td>yağlama</td>
</tr>
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<td>Yuvarlanma direnci</td>
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<td>makara</td>
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